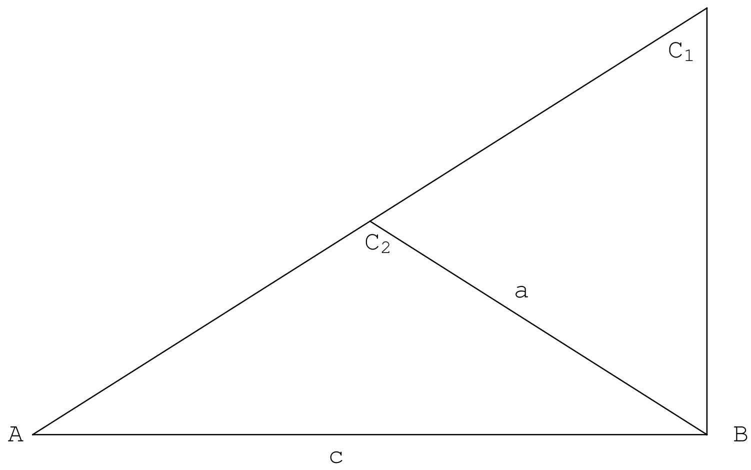
[10-01-24-T11]

Law of Sines (REV)

Suppose two sides and angle not included are given. For example, angle ∠A and sides a and c as shown below.



Apply the Law of Sines to find C_1,

$\frac{\sin C_1}{c} = \frac{\sin A}{a} \Longrightarrow \sin C_1 = (\frac{c}{a}) \sin A$.

Case 1: $\sin C_1 > 1$.
No triangle.

Case 2: $\sin C_1 \leq 1$.

$C_1 = \sin^{-1}[(\frac{c}{a}) \sin A]$

$C_2 = 180° - C_1$

subcase: If $A + C_2 < 180°$, two triangles. They are △ABC$_1$ and △ABC$_2$.

subcase: If $A + C_2 \geq 180°$, one the triangle. It is △ABC$_1$.

[EX1] Find the angles and sides of the triangle ABC such that ∠A = 30°, $a = 10\,m$, $c = 14\,m$.

Solution,

$\frac{\sin 30°}{10} = \frac{\sin C}{14} \Longrightarrow \sin C = \frac{14}{10} \sin 30° = \frac{14}{10} (\frac{1}{2}) = \frac{7}{10}$. Since there are two angles whose sine is $\frac{7}{10}$, there may be two triangles that satisfy the given conditions.

$C = \sin^{-1}(\frac{7}{10}) = 44.427$.

$C_2 = 180 - 44.427 = 135.573$. Since $A + C_2 = 30 + 135.573 = 165.573 < 180$, there *are* two triangles that satisfy the given conditions. They are:

$\triangle ABC_1$ with

∠A = 30 °	a = 10 m
∠B = 105.573	b = 19.27
∠C_1 = **44.427**	14 m

and

$\triangle ABC_2$ with

∠A = 30 °	a = 10 m
∠B = 14.427	b = 4.98
∠C_2 = **135.573**	14 m

The Law of Sines was used to find the lenght of side b in each of the triangles: $b = a\,(\frac{\sin B}{\sin A})$